CBSE Board
Class XI Mathematics
Sample Paper – 10

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Find $\lim_{x\to 0}\frac{3^x-2^x}{x}$.

2. Write contrapositive of the statement: If Mohan is a poet then he is poor.

3. Write the value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$.

OR

Write the value of $\sqrt{-25}\times\sqrt{-9}$.

4. What is the total number of elementary events associated to the random experiment of throwing three dice together?

SECTION – B

5. Let A = {x, y, z} B = {1, 2}, findind the number of relations from A to B.

6. If f(x) = sin [log (x + $\sqrt{x^2+1}$)] then show that f(-x) = -f(x).

OR

If f(x) = $\frac{1+x}{1-x}$ show that f[f(tan Θ)] = -cotΘ.

7. An arc AB of a circle subtends an angle x radians at the centre O of the circle. Given that the area of a sector AOB is equal to the square of the length of the arc AB, find the value of x.

OR

Find the degree measure of $\frac{5\pi}{3}$ and 4π.

8. i. Is the following pair equal? Justify?
A = {x : x is a letter in the word “LOYAL”}, B = {x : x is a letter of the word “ALLOY”}
ii. Is the set C = $\{x : x \in Z \text{ and } x^2 = 36\}$ finite or infinite?

9. In triangle ABC, if a = 3, b = 5 and c = 7 find cosA, cosC.

OR

In triangle ABC, $(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}=c^2$ incomplete question

10. Write converse of the statement “If a number is even then n^2 is even.”

11. Find domain of the function f(x) = $\sqrt{4-x}+\frac{1}{\sqrt{x^2-1}}$

12. Find the centre and radius of a circle : $x^2 + y^2 - 4x + 6y = 12$

SECTION – C

13. Compute sin 75°, cos 75° and tan 15° from the functions of 30° and 45°.

14. If f(x) = $\log\left(\frac{1-x}{1+x}\right)$ show that f(a) + f(b) = $f\left(\frac{a+b}{1+ab}\right)$

15. Find the domain of
i. $\sqrt{x}+\sqrt{2x-1}$
ii. $\log(x-2)-\sqrt{3-x}$

16. The sum of the first three terms of G. P. is 7 and the sum of their squares is 21. Determine the first five terms of the G. P.

17. For any two complex numbers z_1 and z_2 and any real numbers a and b, prove that
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2+b^2)\left[|z_1|^2+|z_2|^2\right]$

18. When two dice are thrown. Calculate the probability of throwing a total of

i. A 7 or an 11

ii. A doublet or a total of 6.

19. Sum up 5 + 55 + 555 + ... to n terms.

20. Find the value of $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$ and show that the value of $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

OR

A code word is consist of two distinct English alphabets followed by two distinct numbers from 1 to 9. For example CA23 is a code word. How many such code words are there? How many of them end with an even integer?

21. Find the equation of the line through the point (4, -5) and parallel to 3x + 4y + 5 = 0 and perpendicular to 3x + 4y + 5 = 0.

OR

The length L (in cm) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

22. (i) Find the derivative of $f(x)=-\frac{1}{x}$, using first principle.

(ii) Evaluate: $\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}$

OR

(i) Find the derivative of the given function using first principle:

$f(x)=\cos\left(x-\frac{\pi}{16}\right)$

(ii) Evaluate: $\lim_{x\to\frac{\pi}{2}}\frac{5^{\cos x}-1}{\frac{\pi}{2}-x}, x\neq\frac{\pi}{2}.$

23. Find the equations of the lines through the point (3, 2) which are at an angle of 45° with the line x − 2y = 3.

SECTION – D

24. If in a ΔABC, $\frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}$,then prove that: $\frac{\cos A}{2}=\frac{\cos B}{7}=\frac{\cos C}{11}$.

OR

If A = $\cos^2\theta+\sin^4\theta$ prove that $\frac{3}{4}\leq A\leq 1$ for all values of θ.

25. Given below is the frequency distribution of weekly study hours of a group of class 11 students. Find the mean, variance and standard deviation of the distribution using the short cut method.

Classes	Frequency
0 - 10	5
10 - 20	8
20 - 30	15
30 - 40	16
40 - 50	6

26. Prove that:

$$\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$$

27. Find the solution region for the following system of inequations:

$x + 2y \le 10, x + y \ge 1, x - y \le 0, x \ge 0, y \ge 0$

OR

Solve the inequality given below and represent the solution on the number line.

$$\frac{1}{2}\left(\frac{3x+20}{5}\right)\ge\frac{1}{3}(x-6)$$

28. The sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m$ is 559, where $x \neq 0$ and m being a natural number. Find the term of the expansion containing x^3.

29. Find the sum of the following series upto n terms:

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+ \dots\dots\dots + \dots\dots\dots$$

OR

If S_1, S_2, S_3 be the sum of n, 2n and 3n terms of a GP respectively.
Prove that $S_1 (S_3 - S_2) = (S_2 - S_1)^2$

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Sample Paper – 10 Solution

SECTION – A

1.

$$\lim_{x\to 0}\frac{3^x-2^x}{x}$$
$$=\lim_{x\to 0}\frac{3^x-1-2^x+1}{x}$$
$$=\lim_{x\to 0}\frac{3^x-1-\left(2^x-1\right)}{x}$$
$$=\lim_{x\to 0}\frac{3^x-1}{x}-\left(\frac{2^x-1}{x}\right)$$
$$=\log 3-\log 2$$
$$=\log\frac{3}{2}$$

2. If Mohan is not poor then he is not a poet.

3.

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$
$$=\frac{i^{584}\left(i^8+i^6+i^4+i^2+1\right)}{i^{574}\left(i^8+i^6+i^4+i^2+1\right)}$$
$$=i^{584-574}$$
$$=i^{10}$$
$$=\left(i^2\right)^5$$
$$=(-1)^5$$
$$=-1$$

OR

$$\sqrt{-25}\times\sqrt{-9}=\sqrt{25}\times\sqrt{-1}\times\sqrt{9}\times\sqrt{-1}=5i\times 3i=15i^2=-15$$

4. The total number of elementary events associated to the random experiment of thrown a dice is 6^n where n is the number of throws. Hence, the total number of elementary events associated to the random experiment of throwing a three dice together is 6^3 = 216.

SECTION – B

5. n(A) = 3, n(B) = 2 ∴ n(A × B) = 3 × 2 = 6

The number of subsets of A × B = 2^6 = 64

The number of relations from A into B = 64

6. f(x) = = sin [log (x + $\sqrt{x^2+1}$)]

f(-x) = sin [log (-x + $\sqrt{x^2+1}$)]

$$= \sin\left[\log\left(\left(\sqrt{x^2+1}-x\right)\times\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right)\right]$$

$$= \sin\left[\log\left(\frac{1}{\sqrt{x^2+1}+x}\right)\right]$$

$$= \sin\left[\log\left(\sqrt{x^2+1}+x\right)^{-1}\right]$$

$$= \sin\left[-\log\left(\sqrt{x^2+1}+x\right)\right]$$

$$= -\sin\left[\log\left(\sqrt{x^2+1}+x\right)\right]$$

= -f(x)

OR

$f(x) = \frac{1+x}{1-x}$ ∴ $f(\tan\theta)=\frac{1+\tan\theta}{1-\tan\theta}$

$$f\left[f(\tan\theta)\right]=\frac{1+\frac{1+\tan\theta}{1-\tan\theta}}{1-\frac{1+\tan\theta}{1-\tan\theta}}$$

$$f\left[f(\tan\theta)\right]=\frac{1-\tan\theta+1+\tan\theta}{1-\tan\theta-(1+\tan\theta)}$$

$$f\left[f(\tan\theta)\right]=\frac{2}{-2\tan\theta}=-\cot\theta$$

7. Taking θ = x we get area of sector AOB = $\frac{1}{2}r^2x$

$$\frac{1}{2}r^2x = s^2$$

$$\frac{1}{2}r^2x = r^2x^2 \qquad \because S = rx$$

$$x = \frac{1}{2}\text{ rad}$$

OR

$$\frac{5\pi}{3} = \frac{5\pi}{3} \times \frac{180}{\pi} = 300° \qquad \because 1^c = \left(\frac{180}{\pi}\right)^\circ$$

$$4\pi = 4\pi \times \frac{180}{\pi} = 720°$$

8. i. A = {L, O, Y, A} and B = {A, L, O, Y}

Clearly A = B

ii. C = {x: $x \in Z$ and x^2 = 36} = {6, -6}

So, C is a finite set.

9. In triangle ABC, if a = 3, b = 5 and c = 7

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 49 - 9}{2 \times 5 \times 7} = \frac{65}{70} = \frac{13}{14}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{9 + 25 - 49}{2 \times 3 \times 5} = \frac{-15}{30} = \frac{-1}{2}$$

OR

$$(a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$$

$$= a^2\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right) + b^2\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right) - 2ab\left(\cos^2\frac{C}{2} - \sin^2\frac{C}{2}\right)$$

$$= a^2 + b^2 - 2ab\cos C$$

$$= c^2$$

10. If a number n^2 is even then n is even.

11. f(x) is defined for all x satisfying

$4 - x \geq 0$ and $x^2 - 1 > 0$

$x - 4 \leq 0$ and $(x-1)(x+1) > 0$

$x \leq 4$ and $x \leq -1$ or $x > 1$

$x \in (-\infty, -1) \cup (1, 4]$

Domain f = $(-\infty, -1) \cup (1, 4]$

12. $x^2 + y^2 - 4x + 6y = 12$

$x^2 - 4x + y^2 + 6y = 12$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

$(x - 2)^2 + (y + 3)^2 = 25$

$(x - 2)^2 + [y - (-3)^2] = 5^2$

Comparing with the equation

$(x - a)^2 + [y - b^2] = r^2$

Radius of the circle is 5 units and centre is (2, -3).

SECTION – C

13. Sin 75° = sin (45° + 30°)

= sin 45°cos 30° + cos 45° sin 30°

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Cos 75° = cos (45° + 30°)

= cos 45° cos 30° – sin 45° sin 30°

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

tan 15° = tan (45° – 30°)

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3-2\sqrt{3}+1}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

14. $f(x)=\log\left(\frac{1-x}{1+x}\right)$

$$f(a)+f(b)=\log\left(\frac{1-a}{1+a}\right)+\log\left(\frac{1-b}{1+b}\right)$$

$$=\log\left(\frac{1-a}{1+a}\times\frac{1-b}{1+b}\right)$$

$$=\log\left(\frac{1-b-a+ab}{1+b+a+ab}\right)$$

$$=\log\left(\frac{1+ab-b-a}{1+ab+b+a}\right)$$

$$=\log\left(\frac{1-\frac{b+a}{1+ab}}{1+\frac{b+a}{1+ab}}\right)$$

$$=f\left(\frac{a+b}{1+ab}\right)$$

$$f(a)+f(b)=f\left(\frac{a+b}{1+ab}\right) \text{ where } f(x)=\log\left(\frac{1-x}{1+x}\right)$$

15. i. $\sqrt{x}+\sqrt{2x-1}$

$f(x)=\sqrt{x}$ and $g(x)=\sqrt{2x-1}$

Let domain of f(x) = A and domain of g(x) = B

Thus, $A=[0,\infty)$ and $B=\left[\frac{1}{2},\infty\right)$

Domain of $\sqrt{x}+\sqrt{2x-1}$ = A ∩ B = $\left[\frac{1}{2},\infty\right)$

ii. $\log(x-2)-\sqrt{3-x}$

$f(x)=\log(x-2)$ and $g(x)=\sqrt{3-x}$

For f(x) to be defined x – 2 > 0 ..reason
x > 2 then $x \in (2, \infty)$ and g(x) to be defined 3 – x ≥ 0
3 ≥ x i. e. x ≤ 3 hence, $x \in (-\infty, 3]$

Domain of $\log(x-2) - \sqrt{3-x}$ = A ∩ B – { x | g(x) = 0}

=A ∩ B – {3}

= (2, ∞) ∩ (-∞, 3) – {3} = (2, 3)

16. Let the first three terms of the G. P. be a, ar, ar^2.

$a + ar + ar^2 = 7$

$a(1 + r + r^2) = 7$............(i)

$a^2 + a^2r^2 + a^2r^4 = 21$......(ii)

$a^2(1 + r^2 + r^4) = 21$

$a^2 (1 + r^2 + r)(1 + r^2 - r) = 21$

$a^2(1 + r + r^2)^2 = 49$ ....from (i)........(iii)

Dividing (ii) by (iii)

$$\frac{1-r+r^2}{1+r+r^2} = \frac{3}{7}$$

$7 - 7r + 7r^2 = 3 + 3r + 3r^2$

$2r^2 - 5r + 2 = 0$

$(2r - 1)(r - 2) = 0$

r = ½ or 2

When r = ½ then a(1 + ½ + ¼) = 7 hence, a = 4

The first five terms of the G. P. are 4, 2, 1 ½, ¼

When r = 2 then a(1 + 2 + 4) = 7 then a = 1

The first five terms of the G. P. are 1, 2, 4, 8, 16.

17. Let $z_1 = x_1 + y_1i$ and $z_2 = x_2 + y_2i$

LHS

$= |a(x_1 + y_1i) - b(x_2 + y_2i)|^2 + |b(x_1 + y_1i) + a(x_2 + y_2i)|^2$

$= |ax_1 - bx_2 + (ay_1 - by_2)i|^2 + |bx_1 + ax_2 + (by_1 + ay_2)i|^2$

$= a^2x_1^2 + b^2x_2^2 - 2abx_1x_2 + a^2y_1^2 + b^2y_2^2 - 2aby_1y_2 +$

$b^2x_1^2 + a^2x_2^2 + 2abx_1x_2 + b^2y_1^2 + a^2y_2^2 + 2aby_1y_2$

$= a^2x_1^2 + b^2x_2^2 + a^2y_1^2 + b^2y_2^2 + b^2x_1^2 + a^2x_2^2 + b^2y_1^2 + a^2y_2^2$

$= \left(a^2 + b^2\right)\left[\left(x_1^2 + y_1^2\right) + \left(x_2^2 + y_2^2\right)\right]$

$= \left(a^2 + b^2\right)\left[|z_1^2| + |z_2^2|\right]$

= RHS

18. i. A : Getting a total of 7 and B : getting a total of 11

A = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

n(A) = 6, P(A) = n(A)/n(S) = 6/36

B = {(5, 6), (6, 5)}

n(B) = 2, P(B) = n(B)/n(S) = 2/36

The two events are mutually exclusive.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$

ii. The sample space consists of 36 sample points

n(S) = 36

A : Getting a doublet

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n(A) = 6

B : getting a total of 6

B = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}

n(B) = 5

P(A) = n(A)/n(S) = 6/36 and P(B) = n(B)/n(S) = 5/36

The two events are not mutually exclusive since (3, 3) is one common sample point.

$P(A \cap B) = P(A \cap B)/n(S) = 1/36$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 6/36 + 5/36 - 1/36$

$= 10/36$

$= 5/18$

19. $S = 5 + 55 + 555 + \ldots + T_{n-1} + T_n$

$S = 5 + 55 + 555 + \ldots T_{n-2} + T_{n-1} + T_n$

Subtracting we get

$5 + 50 + 500 + \ldots + T_n - T_{n-1} - T_n = 0$

$T_n = 5 + 50 + 500 + \ldots$ n terms

$T_n = 5\left(\frac{10^n - 1}{10 - 1}\right)$

$T_n = \frac{5}{9}\left(10^n - 1\right)$

$T_{n-1} = \frac{5}{9}\left(10^{n-1} - 1\right)$

$T_{n-2} = \frac{5}{9}\left(10^{n-2} - 1\right)$

....

$T_2 = \frac{5}{9}\left(10^2 - 1\right)$

$T_1 = \frac{5}{9}\left(10 - 1\right)$

Adding we get

$$S = \frac{5}{9}\left[\left(10+10^2+10^3+....+10^{n-1}+10^n\right)-\Sigma 1\right]$$

$$= \frac{5}{9}\left[\left(\frac{10\left(10^n-1\right)}{10-1}\right)-n\right]$$

$$= \frac{5}{9}\left[\left(\frac{10\left(10^n-1\right)}{9}\right)-n\right]$$

20. $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6=2\left[\left(\sqrt{2}\right)^6+{}^6C_2\left(\sqrt{2}\right)^4+{}^6C_4\left(\sqrt{2}\right)^2+1\right]$

$=2\left(8+\frac{6\times5}{1\times2}\times4+\frac{6\times5}{1\times2}\times2+1\right)$

= 2(8 + 60 + 30 + 1)

= 198

$\left(\sqrt{2}-1\right)^6=(1.42-1)^6=0.42^6<1$

$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6=198$

$\left(\sqrt{2}+1\right)^6=198-\left(\sqrt{2}-1\right)^6$

$\left(\sqrt{2}+1\right)^6=198$ – a number between 0 and 1

$\left(\sqrt{2}+1\right)^6=198$ – a number between 197 and 198

Integral part of $\left(\sqrt{2}+1\right)^6$ is 197.

OR

There are 26 letters in English alphabet.
First two places are to be filled by any two of these 26 letters in ${}^{26}P_2 = 26 \times 25$ ways.
There are 9 distinct numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. Last two places are to be filled by any two of these 9 numbers in ${}^{9}P_2 = 9 \times 8$ ways.
Associating the required number of code words = 26 × 25 × 9 × 8 = 46800
The first two places can be filled in 26 × 25 ways.
Now to end with an even number, the fourth place can be filled by any one out of 2, 4, 6, 8 in 4 ways.
Third place can be filled by any of the remaining 8 numbers in 8 ways.
Thus third and fourth places can be filled in 4 × 8 ways
Associating the required number of code words = 26 × 25 × 4 × 8 = 20800

21. Slope of the line 3x + 4y + 5 = 0 is -3/4 Comparing with y = mx + c.........(i)

The equation of the lines passing through the point (4, -5) and parallel to (i) is

y + 5 = -3/4(x – 4)

4y + 20 = -3x + 12

3x + 4y + 8 = 0

Slope of the line perpendicular to (i) is 4/3

The equation of the line perpendicular to (i) and through (4, -5)

y + 5 = 4/3 (x – 4)

3(y + 5) = 4x – 16

3y + 15 = 4x – 16

4x – 3y – 31 = 0

OR

Assuming celcius C along the x – axis and length L along the y-axis, we have the relation

L = mC + k.........(i)

124.942 = 20m + k..........(ii)

When C = 110, L = 125.134

125.134 = 110m + k........(iii)

Subtracting (ii) from (iii)

0.192 = 90m

m = 0.192/90 = 0.213...wrong answer

125.134 = 110 × 0.213 + k

k = 125.134 – 23.430 = 101.704

L = 0.213C + 101.704

Which express L in terms of C.

22.

(i) Derivative of $f(x) = -\frac{1}{x}$, using first principle

$$f(x) = -\frac{1}{x}$$

$$\Rightarrow f(x+\delta x) = -\frac{1}{x+\delta x}$$

$$\Rightarrow f(x+\delta x) - f(x) = -\frac{1}{x+\delta x} - \left(-\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{x+\delta x}$$

$$\Rightarrow f(x+\delta x) - f(x) = \frac{(x+\delta x) - x}{x(x+\delta x)} = \frac{\delta x}{x(x+\delta x)}$$

$$\Rightarrow \frac{f(x+\delta x) - f(x)}{\delta x} = \frac{1}{\delta x} \cdot \frac{\delta x}{x(x+\delta x)} = \frac{1}{x(x+\delta x)}$$

$$f'(x)=\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=\lim_{\delta x\to 0}\frac{1}{x(x+\delta x)}=\frac{1}{x(x)}$$

$$\Rightarrow f'(x)=\frac{1}{x^2}$$

$$\text{(ii)}\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}=\lim_{x\to 0}\frac{2^x\left(3^x-1\right)-1\left(3^x-1\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{\left(3^x-1\right)\left(2^x-1\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{\left(3^x-1\right)}{x}\lim_{x\to 0}\frac{\left(2^x-1\right)}{x}$$

$$=(\ln 3)(\ln 2)$$

OR

$$\text{(i) } f(x)=\cos\left(x-\frac{\pi}{16}\right)$$

$$f(x+\delta x)=\cos\left(x+\delta x-\frac{\pi}{16}\right)$$

$$f(x+\delta x)-f(x)=\cos\left(x+\delta x-\frac{\pi}{16}\right)-\cos\left(x-\frac{\pi}{16}\right)$$

$$=-2\sin\frac{\left(x+\delta x-\frac{\pi}{16}+x-\frac{\pi}{16}\right)}{2}\sin\frac{\left(x+\delta x-\frac{\pi}{16}-\left(x-\frac{\pi}{16}\right)\right)}{2}$$

$$=-2\sin\frac{\left(2x+\delta x-\frac{\pi}{8}\right)}{2}\sin\frac{\delta x}{2}$$

$$\frac{f(x+\delta x)-f(x)}{\delta x}=-\frac{2\sin\frac{\left(2x+\delta x-\frac{\pi}{8}\right)}{2}\sin\frac{\delta x}{2}}{\delta x}=\frac{\sin\frac{\left(2x+\delta x-\frac{\pi}{8}\right)}{2}\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$=-\lim_{\delta x\to 0}\sin\left(x+\frac{\delta x}{2}-\frac{\pi}{16}\right)\lim_{\delta x\to 0}\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$=-\sin\left(x-\frac{\pi}{16}\right)$$

$$(ii)\lim_{x\to\frac{\pi}{2}}\frac{5^{\cos x}-1}{\frac{\pi}{2}-x}=\lim_{y\to 0}\frac{5^{y}-1}{\frac{\pi}{2}-\cos^{-1}y} \qquad [\text{Let } \cos x=y]$$

$$=\lim_{y\to 0}\frac{5^{y}-1}{\sin^{-1}y}$$

$$=\frac{\lim_{y\to 0}\frac{5^{y}-1}{y}}{\lim_{y\to 0}\frac{\sin^{-1}y}{y}}$$

$$=\frac{\ln 5}{1}$$

$$=\ln 5$$

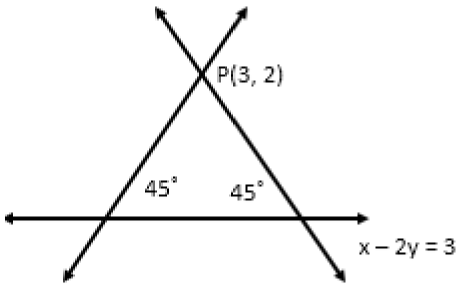
23.

Let the line through (3, 2) be $y - 2 = m(x - 3)$ ... (i)

Slope of line $x - 2y = 3$ is $\frac{1}{2}$.

Now,

$$\tan\left(\pm 45^{\circ}\right)=\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\Rightarrow \pm 1=\frac{2m-1}{2+m}$$



<u>**Case I**</u>: $\frac{2m-1}{2+m}=1\Rightarrow 2m-1=2+m$, so m = 3

Equation of line is $y - 2 = 3(x - 3)$.
Therefore $3x - y - 7 = 0$ is the required equation

<u>**Case II**</u>: $\frac{2m-1}{2+m}=-1\Rightarrow 2m-1=-2-m$, $3m = -1$

$$m = -\frac{1}{3}$$

Now the equation is $y - 2 = -\frac{1}{3}(x-3)$

$3y - 6 = -x + 3$

$x + 3y - 9 = 0$

SECTION – D

24. $\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15}$

$b+c = 12k, c+a = 13k$ and $a+b=15k$

Therefore

$a = 8k$, $b = 7k$ and $c = 5k$

Now

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(7k)^2+(5k)^2-(8k)^2}{2(7k)(5k)} = \frac{10k^2}{70k^2} = \frac{1}{7} = \frac{2}{14}$$

$$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{(5k)^2+(8k)^2-(7k)^2}{2(5k)(8k)} = \frac{40k^2}{80k^2} = \frac{1}{2} = \frac{7}{14}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(8k)^2+(7k)^2-(5k)^2}{2(8k)(7k)} = \frac{88k^2}{112k^2} = \frac{11}{14}$$

Therefore

$\cos A : \cos B : \cos C = 2:7:11$ or $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$

Use the diagram…

OR

$A = \cos^2\theta + \sin^4\theta = \cos^2\theta + (\sin^2\theta)^2$

$-1 \le \sin\theta \le 1$ for all θ

$0 \le \sin^2\theta \le 1$ for all θ

$(\sin^2\theta)^2 \le \sin^2\theta$ for $0 < x < 1, x^n < x$ for all $n \in N - \{1\}$

$\cos^2\theta + (\sin^2\theta)^2 \le \cos^2\theta + \sin^2\theta$ for all θ

$\cos^2\theta + (\sin^2\theta)^2 \le \cos^2\theta + \sin^2\theta$ for all θ

$A \le 1$ for all θ

$A = \cos^2\theta + \sin^4\theta$

$A = 1 - \sin^2\theta + (\sin^2\theta)^2$

$$A = 1 - \frac{1}{4} + \left(\frac{1}{4} - \sin^2\theta + (\sin^2\theta)^2\right)$$

$$A = \frac{3}{4} + \left(\frac{1}{2} - \sin^2\theta\right)^2$$

$$\left(\frac{1}{2} - \sin^2\theta\right)^2 \geq 0 \text{ for all } \theta$$

$$\frac{3}{4} + \left(\frac{1}{2} - \sin^2\theta\right)^2 \geq \frac{3}{4} \text{ for all } \theta$$

$$A \geq \frac{3}{4} \text{ for all } \theta$$

$$\frac{3}{4} \leq A \leq 1 \text{ for all } \theta$$

25. Let assumed mean be a = 25

Classes	f_i	x_i	$y_i = (x - a)/10$	y_i^2	$f_i y_i$	$f_i y_i^2$
0 - 10	5	5	-2	4	-10	20
10 - 20	8	15	-1	1	-8	8
20 - 30	15	25	0	0	0	0
30 - 40	16	35	1	1	16	16
40 - 50	6	45	2	4	12	24
	50				10	68

$$\sum_{i=1}^{n} f_i y_i = 10, \quad \sum_{i=1}^{n} f_i y_i^2 = 68, \quad \sum_{i=1}^{n} f_i = 50, \quad h = 10$$

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} \times h$$

We get, $\bar{x} = 25 + \frac{10 \times 10}{50} = 27$

$$\sigma_x = \frac{h}{N}\sqrt{N \sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}$$

$$\sigma_X = \frac{10}{50}\left[\sqrt{50 \times 68 - (10)^2}\right]$$

$$\sigma_X = \frac{1}{5} \times 10\sqrt{33} = 11.49$$

$$\sigma_X^2 = 132.02$$

So for the given data Mean = 27, Standard Deviation = 11.49 and Variance = 132.02

26.

Consider $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

$$\cos^2 x+\cos^2(x+\frac{\pi}{3})+\cos^2(x-\frac{\pi}{3})$$

$$=\cos^2 x+\cos^2(x+\frac{\pi}{3})+1-\sin^2(x-\frac{\pi}{3})$$

$$=1+\cos^2 x+\left[\cos^2(x+\frac{\pi}{3})-\sin^2(x-\frac{\pi}{3})\right]$$

$$=1+\cos^2 x+\left[\cos(x+\frac{\pi}{3}+x-\frac{\pi}{3})\cos(x+\frac{\pi}{3}-x+\frac{\pi}{3})\right] \quad \left[\because \cos^2 A-\sin^2 B=\cos(A+B)\cos(A-B)\right]$$

$$=1+\cos^2 x+\cos(2x)\cos\left(\frac{2\pi}{3}\right)$$

$$=1+\cos^2 x+\cos(2x)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2 x+\left(2\cos^2 x-1\right)\left(-\frac{1}{2}\right)$$

$$=1+\cos^2 x+\left(-\cos^2 x+\frac{1}{2}\right)$$

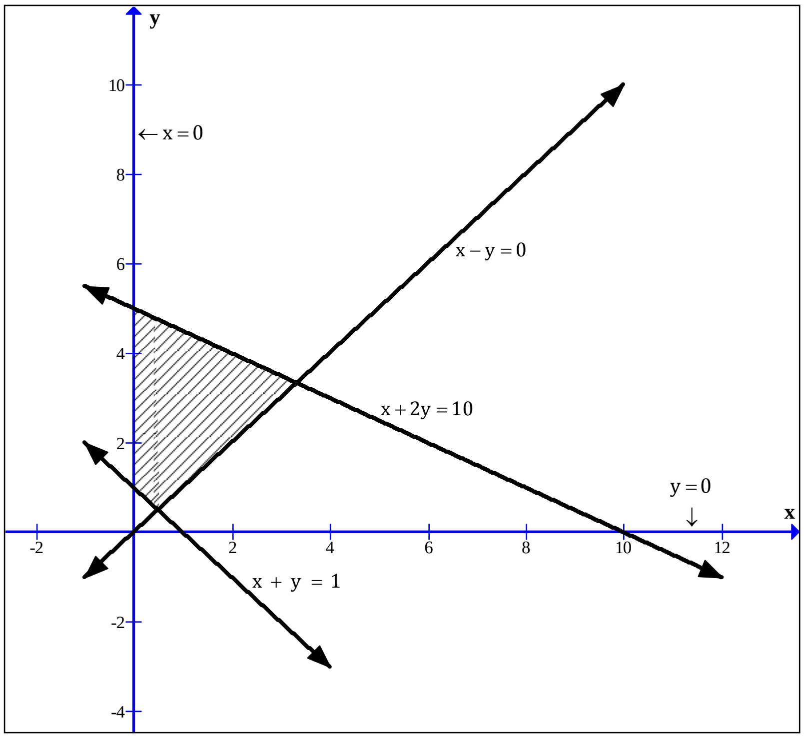
$$=1+\frac{1}{2}=\frac{3}{2}$$

27.

Given inequalities:

$x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0,$

Consider the corresponding equations $x + 2y = 10$, $x + y = 1$ and $x - y = 0$.
On plotting these equations on the graph, we get the graph as shown.
Also we find the shaded portion by substituting (0, 0) in the in equations.



OR

$$\frac{1}{2}\left(\frac{3x+20}{5}\right)\geq\frac{1}{3}(x-6)$$

$$\Rightarrow\frac{1}{10}(3x+20)\geq\frac{1}{3}(x-6)$$

$$\Rightarrow\frac{30}{10}(3x+20)\geq\frac{30}{3}(x-6)$$

$$\Rightarrow 3(3x+20)\geq 10(x-6)$$

$$\Rightarrow 9x+60\geq 10x-60$$

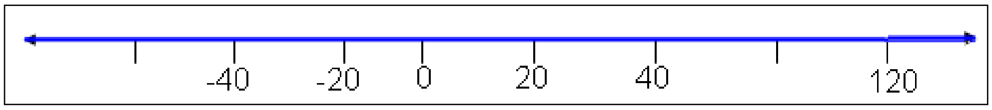
$$\Rightarrow 60+60\geq 10x-9x$$

$$\Rightarrow 120\geq x$$

$$\Rightarrow x\leq 120$$

$$\Rightarrow x\in(-\infty,120]$$

Thus, all real numbers less than or equal to 120 are the solution of the given inequality. The solution set can be graphed on a real line as shown.



$$\Rightarrow 25\% < \frac{1125 \times \frac{45}{100}}{1125 + x} < 30\%$$

$$\Rightarrow \frac{25}{100} < \frac{1125 \times \frac{45}{100}}{1125 + x} < \frac{30}{100}$$

$$\Rightarrow \frac{25}{100} < \frac{1125 \times 45}{(1125 + x) \times 100} < \frac{30}{100}$$

$$\Rightarrow 25 < \frac{1125 \times 45}{(1125 + x)} < 30$$

$$\Rightarrow \frac{1}{25} > \frac{(1125 + x)}{1125 \times 45} > \frac{1}{30}$$

$$\Rightarrow \frac{1125 \times 45}{25} > (1125 + x) > \frac{1125 \times 45}{30}$$

$$\Rightarrow \frac{50625}{25} > (1125 + x) > \frac{50625}{30}$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 - 1125 > x > 1687.5 - 1125$$

$$\Rightarrow 900 > x > 562.5$$

$$\Rightarrow 562.5 < x < 900$$

So the amount of water to be added must be between 562.5 to 900 lt

28. $\left(x - \frac{3}{x^2}\right)^m = {}^mc_0 x^m + {}^mc_1 x^{m-1}\left(\frac{-3}{x^2}\right) + {}^mc_2\, x^{m-2}\left(\frac{-3}{x^2}\right)^2 + \dots\dots\dots + \left(\frac{-3}{x^2}\right)^m$

Coefficient of first 3 terms are: ${}^mc_0, {}^mc_1(-3)^1, {}^mc_2(-3)^2$

So ${}^mc_0 - 3\,{}^mc_1 + 9\,{}^mc_2 = 559$

$1 - 3m + 9\frac{m(m-1)}{2} = 559$

$\Rightarrow 9m^2 - 15m - 1116 = 0$

$3m^2 - 5m - 372 = 0$

$(m - 12)(3m + 31) = 0$

$m = 12, \frac{-31}{3}$ rejecting $(-)^{ve}$ sign

Now $T_{r+1} = {}^{12}c_r\,(x)^{12-r}\left(\frac{-3}{x^2}\right)^r$

For coefficient of x^3, $12 - 3r = 3 \Rightarrow r = 3$

$T_{3+1} = {}^{12}C_3(-3)^3 x^3$ Hence, Required term = $T_4 = -5940x^3$

29. k^{th} term of the given series

$$T_k = \frac{1^3+2^3+3^3+...+k^3}{1+3+5+...+(2k-1)}.$$

$$= \frac{\left[\frac{k(k+1)}{2}\right]^2}{\frac{k}{2}[2+2(k-1)]}$$

$$= \frac{k^2(k+1)^2}{4} \times \frac{2}{2k^2}$$

$$= \frac{(k+1)^2}{4}$$

$$\Sigma t_k = \frac{1}{4}\left[\Sigma k^2 + \Sigma 2k + \Sigma 1\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + 2\,\frac{n(n+1)}{2} + n\right]$$

$$= \frac{n}{24}\ [(n+1)(2n+1) + 6(n+1) + 6]$$

$$= \frac{n}{24}\ (2n^2 + 9n + 13)$$

OR

Let first term of the G.P. be a and common ratio be r

$$S_1 = \frac{a(r^n-1)}{r-1}$$

$$S_2 = \frac{a(r^{2n}-1)}{r-1}$$

$$S_3 = \frac{a(r^{3n}-1)}{r-1}$$

$$\text{L.H.S.} = \frac{a(r^n-1)}{r-1}\left[\frac{a(r^{3n}-1)}{r-1} - \left(\frac{a(r^{2n}-1)}{r-1}\right)\right]$$

$$= \frac{a(r^n-1)}{r-1} \times \frac{a\left[r^{3n}-1-r^{2n}+1\right]}{r-1}$$

$$= \frac{a^2\,(r^n-1)}{r-1} \times r^{2n}\,\frac{(r^n-1)}{r-1}$$

$$= a^2\,r^{2n}\,\frac{(r^n-1)^2}{(r-1)^2}$$

$$= \left[\frac{ar^n(r^n - 1)}{r - 1}\right]^2$$

$$= \left[\frac{a(r^{2n} - r^n)}{r - 1}\right]^2$$

$$= \left[\frac{a\left[\left(r^{2n} - 1\right) - \left(r^n - 1\right)\right]}{r - 1}\right]^2$$

$$= \left[\frac{a(r^{2n} - 1)}{r - 1} - \frac{a(r^n - 1)}{r - 1}\right]^2$$

$= (S_2 - S_1)^2 =$ R.H.S.